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HOMEOSTATIC TENDENCIES OF FIRM-SIZE DISTRIBUTIONS AND THE EVOLUTION OF ECONOMIC SYSTEMS

STABILITE DES DISTRIBUTIONS DE TAILLES D'ENTREPRISES ET EVOLUTION DES SYSTEMES ECONOMIQUES

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Abstract : Stable skewed distributions of subsystem-sizes are observed in a variety of biological, sociological and economic systems (Zipf's law, rank-size rule, Pareto-law). Based on a fundamental mathematical property of Pareto distributions a theoretical explanation is proposed, which applies to hierarchically organized systems in general. As supporting evidence the analysis of empirical data concerning the evolution of firm-size distributions of the 500 largest U.S. industrial corporations over the last 25 years is presented.

Résumé : Des distributions stables du type Paretien sont fréquemment observées pour les tailles de sous-systèmes en économie, en urbanisme, en linguistique, en biologie (loi de Zipf, loi rang-taille, loi de Pareto). Nous proposons une explication théorique basée sur une propriété mathématique fondamentale des distributions de Pareto, qui s'applique de façon générale à des système ayant une structure hiérarchique. A l'appui, nous présentons une analyse empirique de données concernant l'évolution de la taille des 500 plus grandes entreprises industrielles américaines durant les 25 dernières années.

Key Words : HIERARCHICAL SYSTEMS, FIRM-SIZE DISTRIBUTIONS, ZIPF'S LAW, PARETO LAW

Mots clés : SYSTEMES HIERARCHIQUES, DISTRIBUTIONS DE TAILLE D'ENTREPRISES, LOI DE ZIPF, LOI DE PARETO

1. *Introduction*

Numerous articles have been concerned with a class of skew distribution functions that appear in a wide range of empirical data describing economic, sociological, biological and symbol systems (Simon 55). Simon 58 again emphasized the need to accumulate a body of knowledge about these regularities and the processes that generate them.

In this paper we propose an answer to the following two questions :

(i) Why do we observe Paretian size-distributions in such a large variety of systems ? and

(ii) Why do these size-distributions remain self-similar during the evolution of a system ?

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2. *Empirical determination of size-distributions*

In all areas of application (economics, geography, biology, linguistics) the empirical determination of size-distributions is based on three hierarchical levels of observation/description of a system :

level 3 : the system is considered as a whole (fat circle in fig. 1, e.g. a country, a national economy).

level 2 : the system is considered as being composed of mutually exclusive subsystems (dotted circles in fig. 1, e.g. cities, firms).

level 1 : the system is considered as being composed of elementary elements (points in fig. 1, e.g. human inhabitants, employees or dollars of asset value).

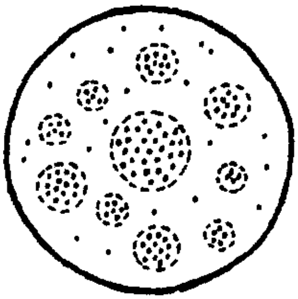

figure 1 : Three level description of a system

The size of a subsystem at a given time t is defined as the number of elementary elements contained within the subsystem. One determines the sizes of all subsystems contained within the system and orders the subsystems according to decreasing size. Thus if the system contains M subsystems, one obtains a sequence of sizes S_1 S_2 ... S_j ... S_M. The index j in the ordered sequence is called rank. Plotting the ranks j as ordinate and the sizes S as abscissa one obtains a distribution called rank-size distribution S(j), which is equivalent to the cumulative frequency distribution commonly used in statistics.

3. *Statistical distributions approximating empirical size data*

The most simple mathematical expression approximating empirical rank-size distributions is called Zipf's law :

$$S(j) = a\, j^{-b} \qquad (1)$$

with S(j) the size of the subsystem ranked j, a and b parameters ; the parameter b corresponding to the slope of the regression line in log-log coordinates.

In the case of a cumulative frequency vs. size representation the most simple mathematical expression approximating empirical data is called Pareto law of the first type :

$$P(s) = \left(\frac{s_o}{s}\right)^{\alpha} \qquad \text{when} \quad s > s_o \qquad (2)$$

with $P(s) = P(S \geq s)$ the percentage of subsystems whose size S exceeds the size s, the parameter α corresponding to the slope of the regression line in log-log coordinates and s_o being a threshold size above which the law is applicable.

Besides these two very simple mathematical laws a whole panoply of more or less complicated distribution functions using more than two parameters have been proposed for fitting the observed data. Without going into details we cite only a few of them : the Pareto law of the second type, the Pareto law of the third type, a modified Zipf's law, the Yule distribution, the lognormal distribution, the Benini law and several composite distributions. In a comparative study fitting a variety of empirical size-distributions by eight different mathematical functions Quandt 64 comes to the conclusion, that all the proposed laws are very similar, and if one law yields a good adjustment of the data, then this is also the case for several other laws. In our view searching for best fits does not lead any further in our comprehension of the observed regularities ; a choice among alternative models can only be made by analyzing and judging the basic theoretical assumptions concerning the underlying processes which lead to the observed distributions.

4. *Theoretical interpretation of rank-size distributions*

4.1. *Panoply of existing models*

The size-distributions of cities have been studied quite extensively and a great number of different models has been proposed by various authors. A very full bibliography containing more than 200 references can be found in a recent study by Pumain 82. Static models "explain" the observed regularities in terms of a spatial economic hierarchy, as an economic optimum, as a state of maximum entropy or as a stationary state of a Markov-process. Constant growth models are based on a process of allometric growth, on a simple stochastic process of redistribution, on a proportional effect of many small changes or on a simple deterministic growth process. Finally in the dynamic models (e.g. Roehner 83) variable growth rates are taken into account.
For firm-size distributions the proposed models are less numerous and all of them have an isomorphic equivalent among the city-size models (Zipf 48, Adelman 58, Steindl 65, Ijiri 74). Summarizing we can say about this panoply of models :

- All proposed models arrive - at least in special cases - at a Pareto law, which is empirically most frequently observed.
- Those models do not have any predictive power in the sense that, for instance, the slope of the distribution for a given country cannot be forecast from other known economic or social variables of that country.

4.2. *An alternative theoretical explanation for the regularities of size-distributions within hierarchically organized systems*

The stability of the Gaussian law is a well known property to every experimental scientist ; i.e. if one observes one Gaussian distribution G' and a second Gaussian distribution G" then the combined distribution $G' \oplus G'' = G$ is Gaussian. (The sign $\oplus$ designates the addition of random variables). The Gaussian probability law is said to be "stable" (under addition). Up to a scale transformation, and up to the choice of the origin, G shows the same properties as G' and G".
The family of all laws which satisfy the stability requirement has been constructed by Lévy 37, Gnedenko 54 : it includes, naturally the Gaussian law, but it includes also certain non-Gaussian laws, each of which turns out to satisfy the Pareto law

$$P(s) \underset{s \to \infty}{\sim} \left(\frac{s_0}{s}\right)^{\alpha} \quad \text{with } 0 < \alpha \leq 2, \text{ or with } 1 < \alpha \leq 2 \quad \text{if the ex-}$$

pected value E(S) is finite. Hence, according to Mandelbrot 60, if a sum of many components is not Gaussian, is skewed, then the most reasonable assumption concerning this sum is that it follows a Pareto law (for large values of the variable).
Looked at from this point of view the observed Pareto distributions loose much of their mystery.
In a static description a Pareto distribution on a given level of observation can be explained as weighted sum of (in a first approximation) independent variables on the next lower level of observation, with each variable being itself distributed according to a Pareto law. In the case of city-size distributions, each city can be considered as an agglomerate of economic units

of different types (barber shops, cleaning, auto-repair, manufacturing, bank, insurance, utilities, transportation enterprises, etc.). According to empirical evidence the size distributions of enterprises of a given type follow a Pareto law within a national economic system (Zipf 47, Simon 58). So following our above arguments it is not surprising, that cities considered as agglomerates of Paretian variables follow a Pareto law.

Similar arguments hold for the distribution of firm-sizes. We consider a business firm as an agglomerate of production units of different types. According to empirical evidence (Simon 58) the size-distributions of production units of a given type follow a Pareto law within a national economy and it is not surprising, that firms considered as agglomerates of Paretian variables follow a Pareto law.

In order to apply our arguments to hierarchically organized systems in general it is sufficient to show :

a) the decomposability of sub-systems into sub-sub-systems on a next lower level of observation/description and

b) the Paretian behaviour of the size distributions of the different types of sub-sub-systems.

So far we have proposed a recursive argument answering question (i). The observed self-similarity of the Paretian distribution during the evolution of a system still remains to be explained.

We consider a Paretian size distribution $S_t(j)$ at time t. The size distribution $S_{t+1}(i)$ at time t+1 shall be generated by the following process :

$$S_{t+1}(i) = \gamma S_t(j) \oplus \Delta(j) \qquad (3)$$

$$\text{with} \quad \Delta(j) = S_{t+1}(j) - \gamma S_t(j)$$

i.e. the resulting size of a subsystem S_{t+1} depends on its initial size S_t multiplied by a constant mean growth factor γ plus a deviation from the mean growth, Δ, added at random. In the case of the evolution of firm-sizes the distribution of the deviations Δ from the mean growth turns out to be Paretian with a parameter α identical to the parameter α of the initial Paretian size distribution (see fig. 2).

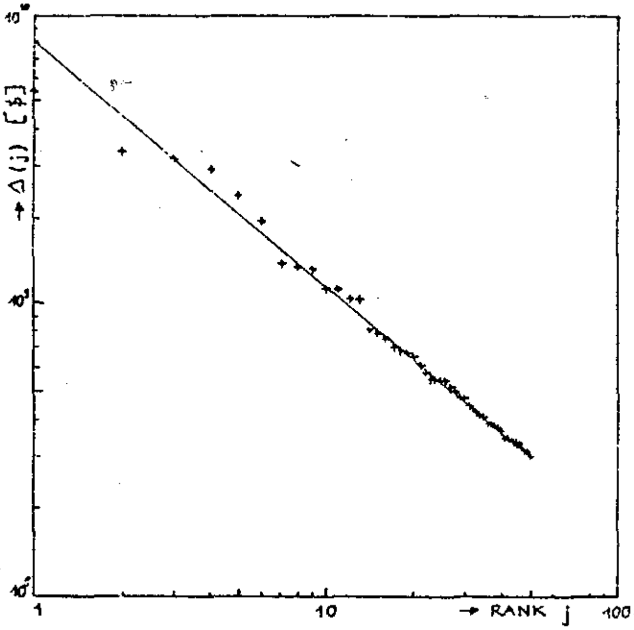

figure 2 : Ranked deviations from mean growth in asset size of the largest U.S. industrial corporations 1981-82.

According to the mathematical property of stability (under addition of Paretian variables one can therefore conclude, that the resulting distribution S_{t+1} of the above process must be Paretian. Since the distributions of both variables S_t and Δ have the same slope α, the slope of the resulting distribution S_{t+1} tends to be identical. Applying process (3) repeatedly leads therefore to a selfsimilarity of the size-distributions during the evolution of the system.

5. *Empirical analysis of the evolution of firm-size distributions*

Unlike the case of city-size distributions, empirical studies concerning firm-size distributions are rather scarce (Zipf 48, Hart 56, Adelman 58, Simon 58, Ijiri 74, Winiwarter 83). A possible explanation for this rather mild interest is the fact that classic economic theory has little to say on the subject. In the following we summarize the results of an empirical analysis concerning the evolution of the 500 largest U.S. industrial corporations during the last 25 years based on the Fortune data.

- The particular form of firm-size distributions within a given economic system reveals, aside from a scale factor, relatively constant over a long period of time : $S_{t+1}(j) \simeq \gamma S_t(j)$, where $S_t(j)$ stands for the size distribution in year t, $S_{t+1}(j)$ for the size distribution in year t+1, and γ a scale factor. This self-similarity of the distribution curves holds in periods of overall economic growth as well as in periods of economic recession despite the fact, that changes in size for a given firm are much more frequent and important than in the case of cities. To illustrate these drastic changes : from the 50 largest industrial firms in 1954 only 20 can be found among the 50 largest in 1982 ; the other 30 have declined in size, been absorbed in mergers or have simply gone out of business. On the other hand 12 of the 50 largest firms in 1982 were not even ranked among the 500 largest in 1954 or simply did not exist at that time. To observe a constant form of the size-distributions despite this intensive shuffling around within the system seems to us quite remarkable. Figure 3 illustrates the asset-size distributions of the 150 largest U.S. industrial corporations for three different years of reference.

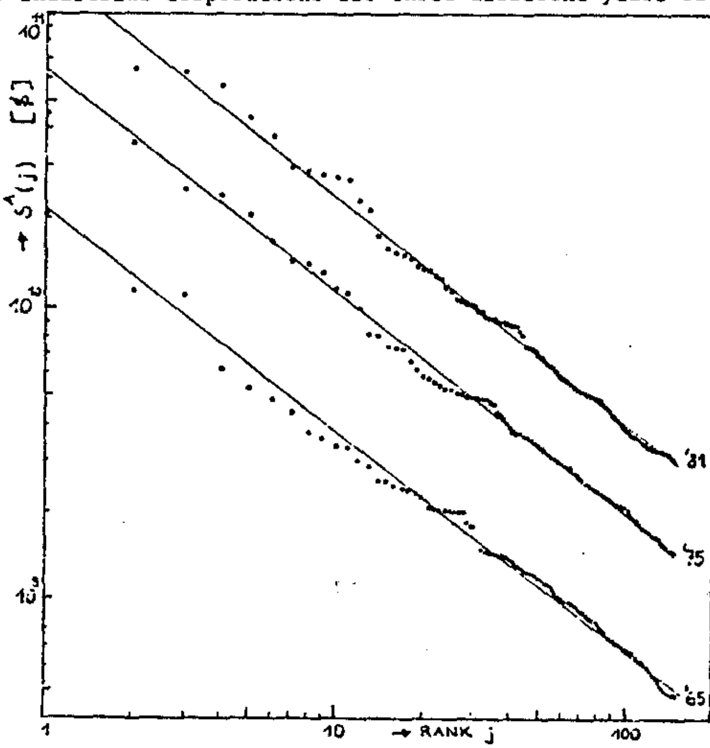

figure 3 : Ranked asset-size distribution of the largest U.S. industrial firms

- A second interesting observation concerns concave or convex departures from the straight line fits to the distributions in log-log coordinates. These departures appear and disappear in time with periods of several years. Departures from the "straight" distribution are rapidly brought "in line" again. For this reason we have chosen the term "homeostatic tendencies" in the title of this paper.
- A third observation concerns the comparison of rank-size distributions using different elementary measures for the firm-size. The following distributions were compared :

$S^A(j)$ ranking firms by the size of their <u>assets</u> in dollars

$S^S(j)$ ranking firms by the size of their total annual <u>sales</u>, and

$S^P(j)$ ranking firms by the size of their total annual net <u>profit</u>.

For a given year the following relations hold between the three different distributions :

$$S^S(j) \simeq \lambda\, S^A(j) \quad \text{and} \quad S^P(j) \simeq \mu S^S(j) \text{ with } \lambda \text{ and } \mu \text{ scale factors,}$$

or stated in other terms $S^P(j) \propto S^S(j) \propto S^A(j)$.
Intuitively we are not surprised to find, that profits are proportional to sales, and sales are proportional to assets within a given industry. What surprises, is that one and the same scale factor applies for the entire distribution, i.e. for all values of j.

6. *Conclusion*

In our view the evolution of firm-size distributions is only a particular case exhibiting regularities of global system behaviour common to self-organizing systems in which processes of birth and death as well as migrations on several hierarchical levels take place.
Based on the fundamental mathematical property of stability (under addition) of Paretian variables we attribute the frequent encounter of Pareto distributions of subsystem sizes to the hierarchical organization of the observed system. Each subsystem can be considered as an agglomerate of different types of sub-sub-systems on the next lower level of observation/description with the size-distributions of each type of sub-sub-system following a Pareto law.
Using the same argument of stability under addition for Paretian variables we attribute the self-similarity of size-distributions during the evolution of a system to the Paretian behaviour of the deviations from mean growth.

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